Separating the Wheat from the Chaff: Efficiently Modeling Thick Tails

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Minimum α

Maximum Eigenvalue

Why?

Research Programme

Accurately and robustly estimate tail probabilities from an economic capital model specification, without simulating

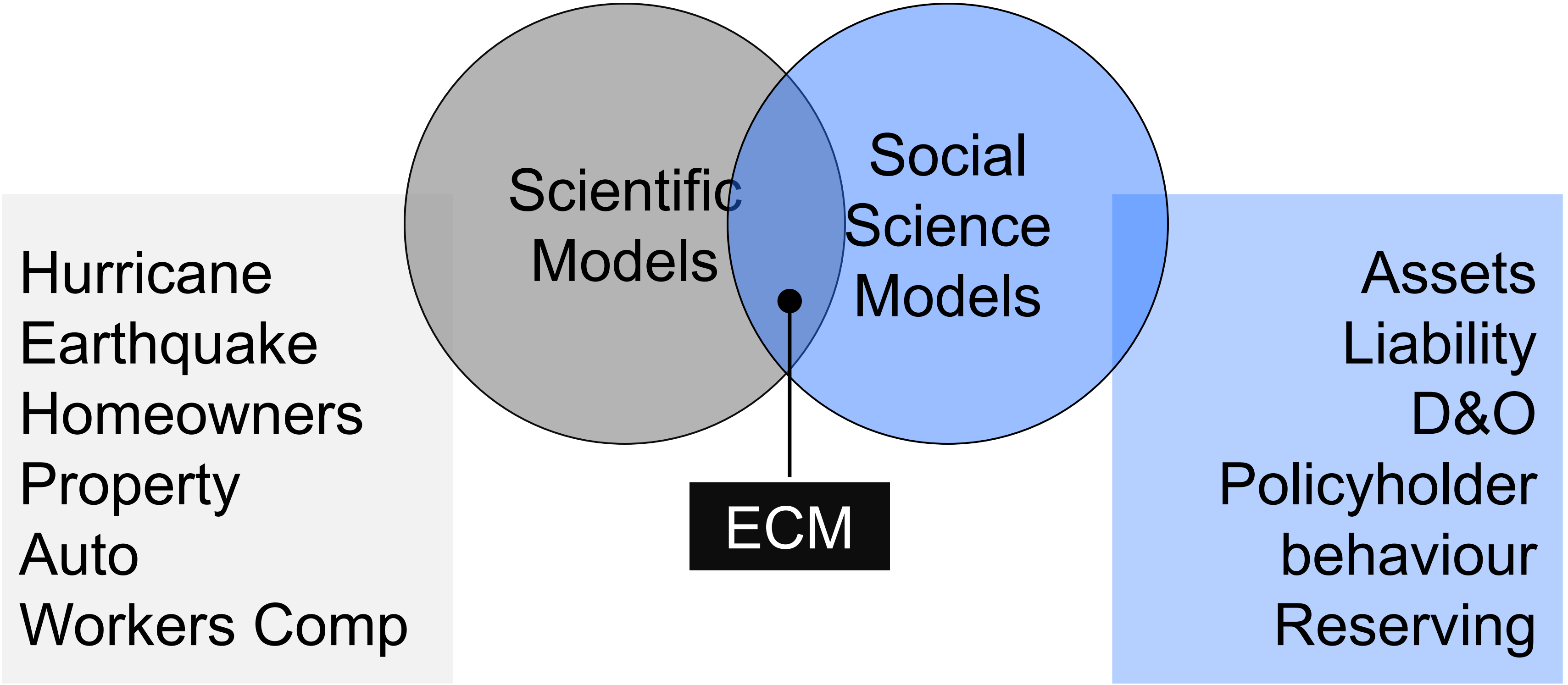
- A supplement to detailed ECM, not a substitute

- Why is programme useful?
  - Reality check and test on detailed model output
    - Does answer have the right order of magnitude?
  - Identify key risk drivers: separate model wheat from model chaff
    - Understanding tail drivers
    - Understand sensitivity of key parameters
  - Improve *communicability* of model results

- And the end of all our exploring / Will be to arrive where we started / And know the place for the first time.

- Talk is a report on work in progress…

ECM Lies at Junction of Two Modeling Worlds



- Economic capital modeling is so difficult because it combines scientific modeling with social science modeling
  - Fixed, repeatable experiments with consistent outcomes
  - No experiments, system adapts to current state of art, “atoms push back”

What Drives Thick Tails in the Aggregate?

Minimum α

Find the Thickest Tail

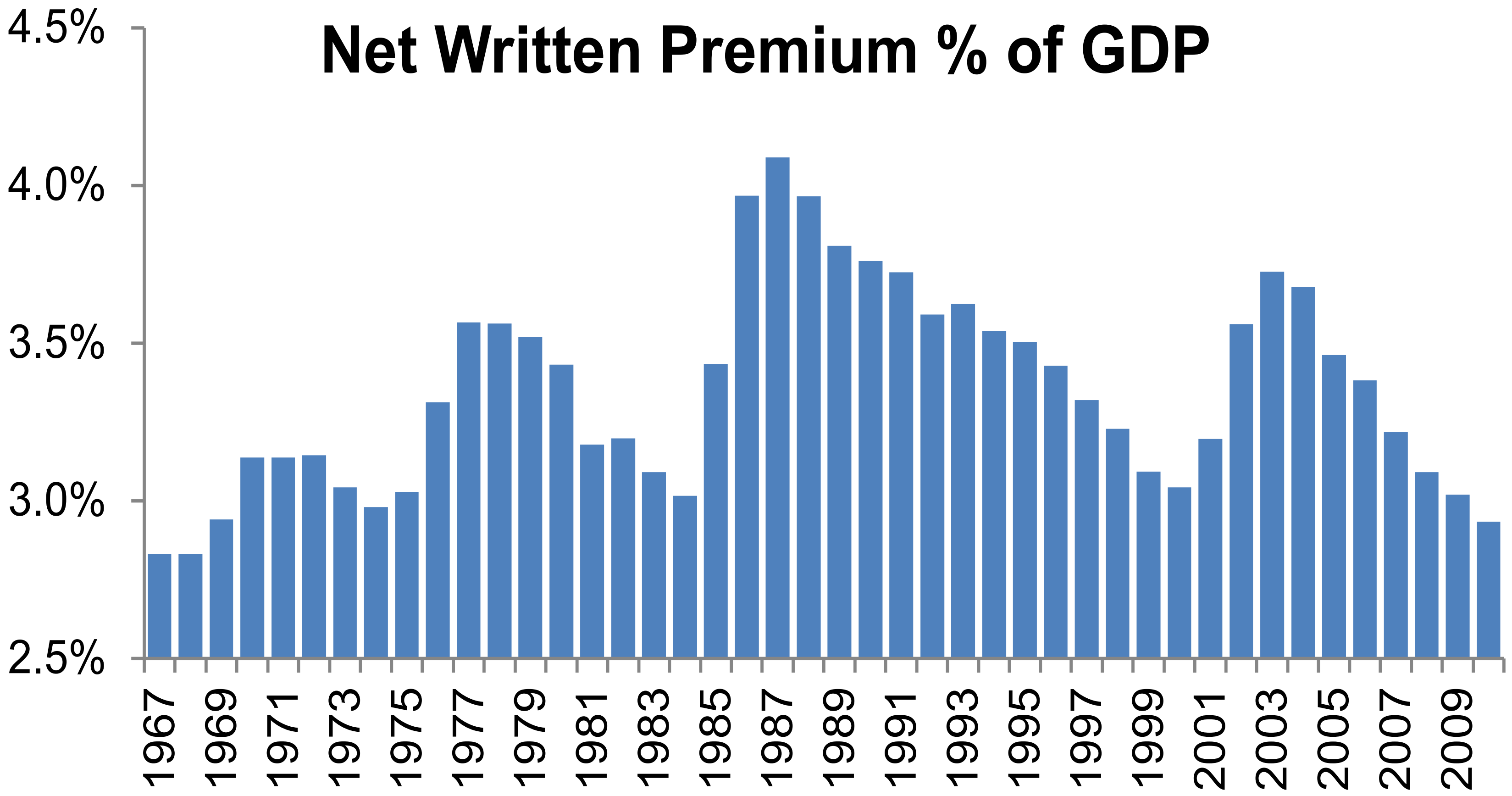
Maximum Eigenvalue

Don’t use Big Correlation Matrices

Why?

Understanding Behaviour Drives Behaviour

Behaviourism in Management: Evidence from the Cycle



Source: A.M. Best Aggregates and Averages, Calendar Year Statutory NWP; BEA GDP

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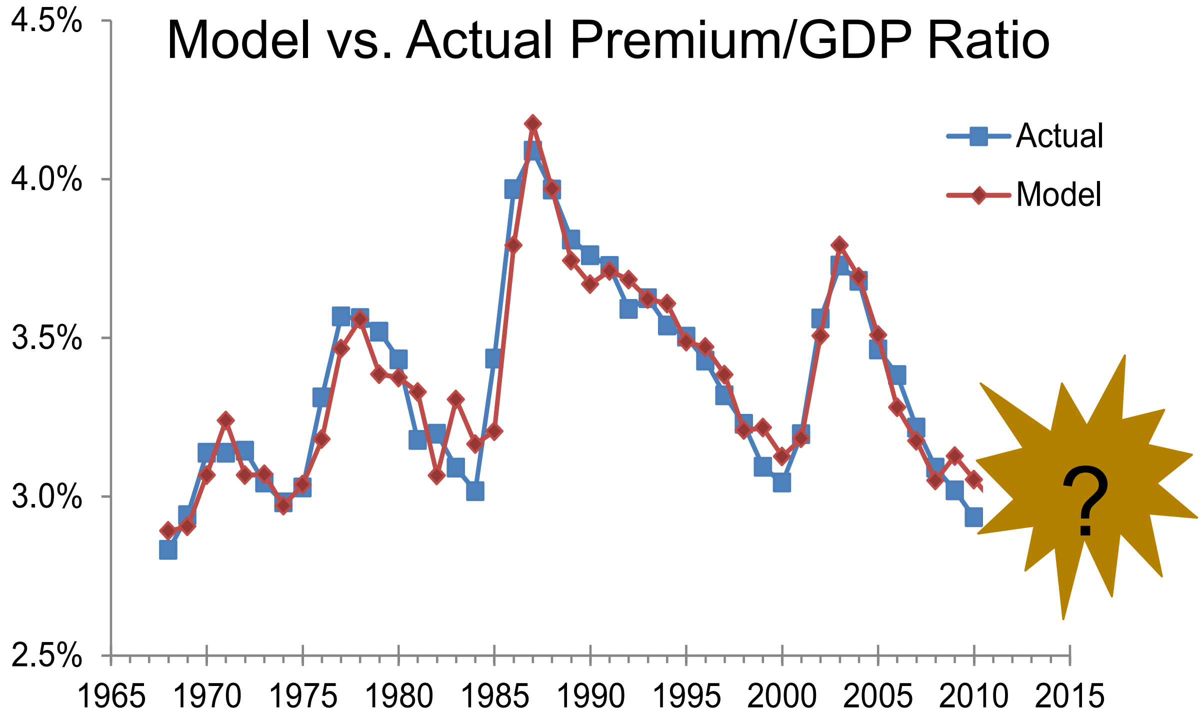
Cycle Dynamics Are Loss Driven With Management Noise



Cycle Model Statistics

	Parameter	Std. Error	p Value
Constant	0.010	0.0017	7.9E-07
Prior Loss/GDP	0.426	0.0798	4.4E-06
Prior Premium/GDP	1.053	0.1149	1.2E-08
Second Prior Premium/GDP	-0.673	0.0936	2.8E-11
Regression R^2	90.9%		

Signal

Noise

Source: Aon Benfield Analytics

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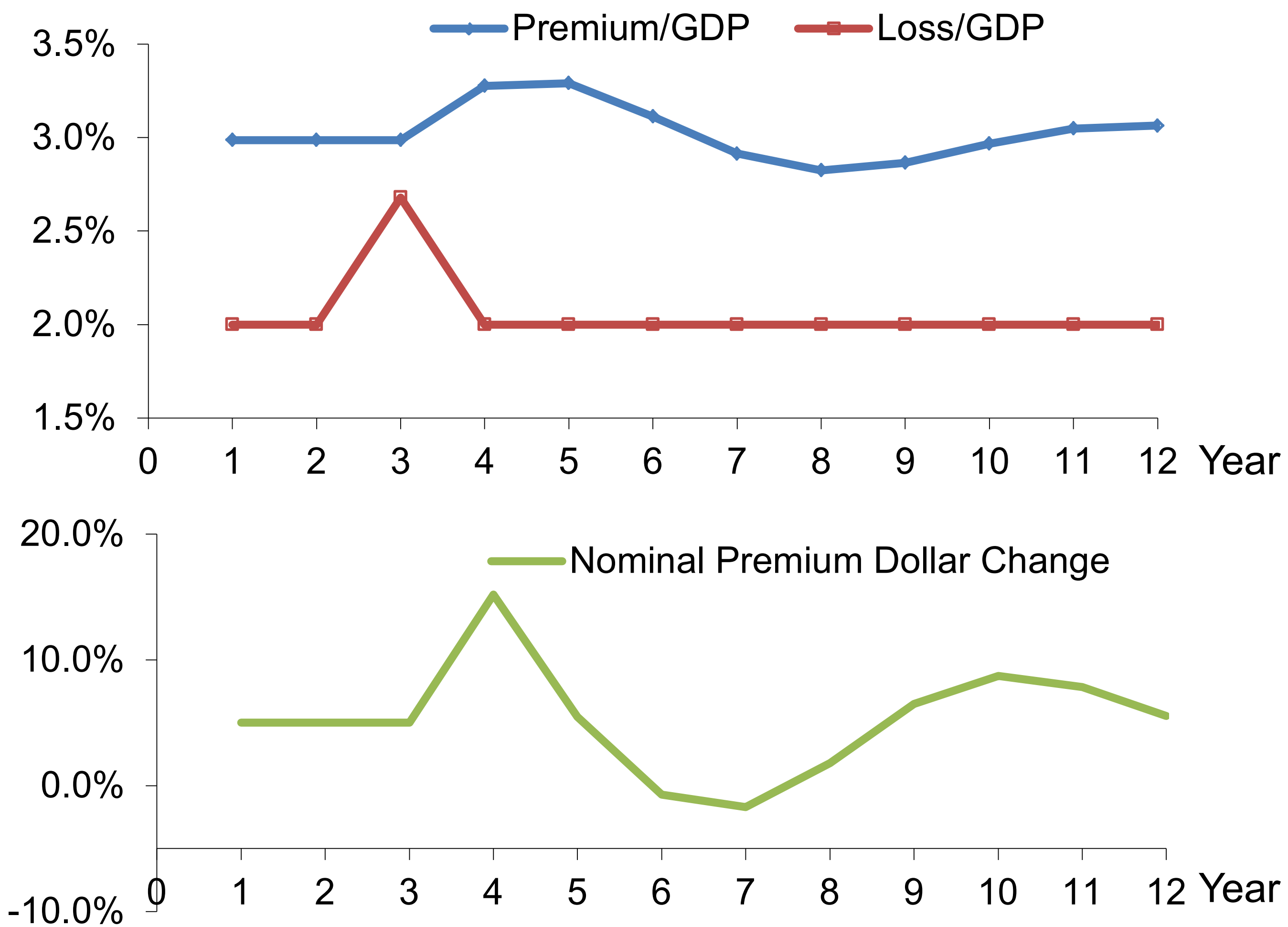
Pro-cyclical Dynamics: Animal Spirits Trump Actuarial Models



- Impact of $100B loss shock on cycle model shows pro-cyclical nature
- Assumes 3% real GDP growth and 2% inflation

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Material Drivers: Impact of Square Root Rule

Unbalanced triangle: change in smallest side length does not change length of diagonal materially

Change in longest side length increases length of diagonal almost 1:1

Balanced triangle: change in one side length produces approx. 70.7% increase in length of diagonal materially

Material Drivers in QIS 5 SCR

Solvency II QIS SCR Sensitivities

Factor	Capital Charge	% Total	BSCR reduction without factor	Pct Reduction	Sensitivity*	Marginal Sensitvity**
Market	36,422	19.5%	106,314	-14.2%	0.48	0.60
Default	2,233	1.2%	122,689	-1.0%	0.57	0.58
Life	44,000	23.6%	111,118	-10.4%	0.29	0.45
Health	10,649	5.7%	121,715	-1.8%	0.21	0.25
Non-Life Non Cat	32,079	17.2%	113,050	-8.8%	0.34	0.84
Non-Life Man Made Cat	10,000	5.4%	123,474	-0.4%	0.05	0.10
Windstorm	65,000	34.8%	92,834	-25.1%	0.48	0.71
Flood	20,000	10.7%	118,646	-4.3%	0.27	0.36
Earthquake	25,000	13.4%	120,839	-2.5%	0.13	0.25
Hail	10,000	5.4%	121,852	-1.7%	0.21	0.26
Subsidence	2,000	1.1%	123,948	0.0%	0.01	0.02
Non-Life Nat Cat	79,555	42.6%	82,071	-33.8%	0.53	0.80
Non-Life Cat	80,181	42.9%	79,981	-35.5%	0.55	0.79
Non-Life	93,510	50.1%	68,645	-44.6%	0.59	0.84
Intangibles	0	0.0%				
Total	186,814	100.0%				
Covariance Diversification	62,847	-33.6%				
BSCR	**123,968**	66.4%				
Adj	0					
Operational	3,000					
SCR	**126,968**					

* 1 EU reduction in factor reduces BSCR/SCR by indicated amount w hen w hole charge is eliminated

** Marginal 1 EU reduction in factor reduces BSCR/SCR by indicated amount

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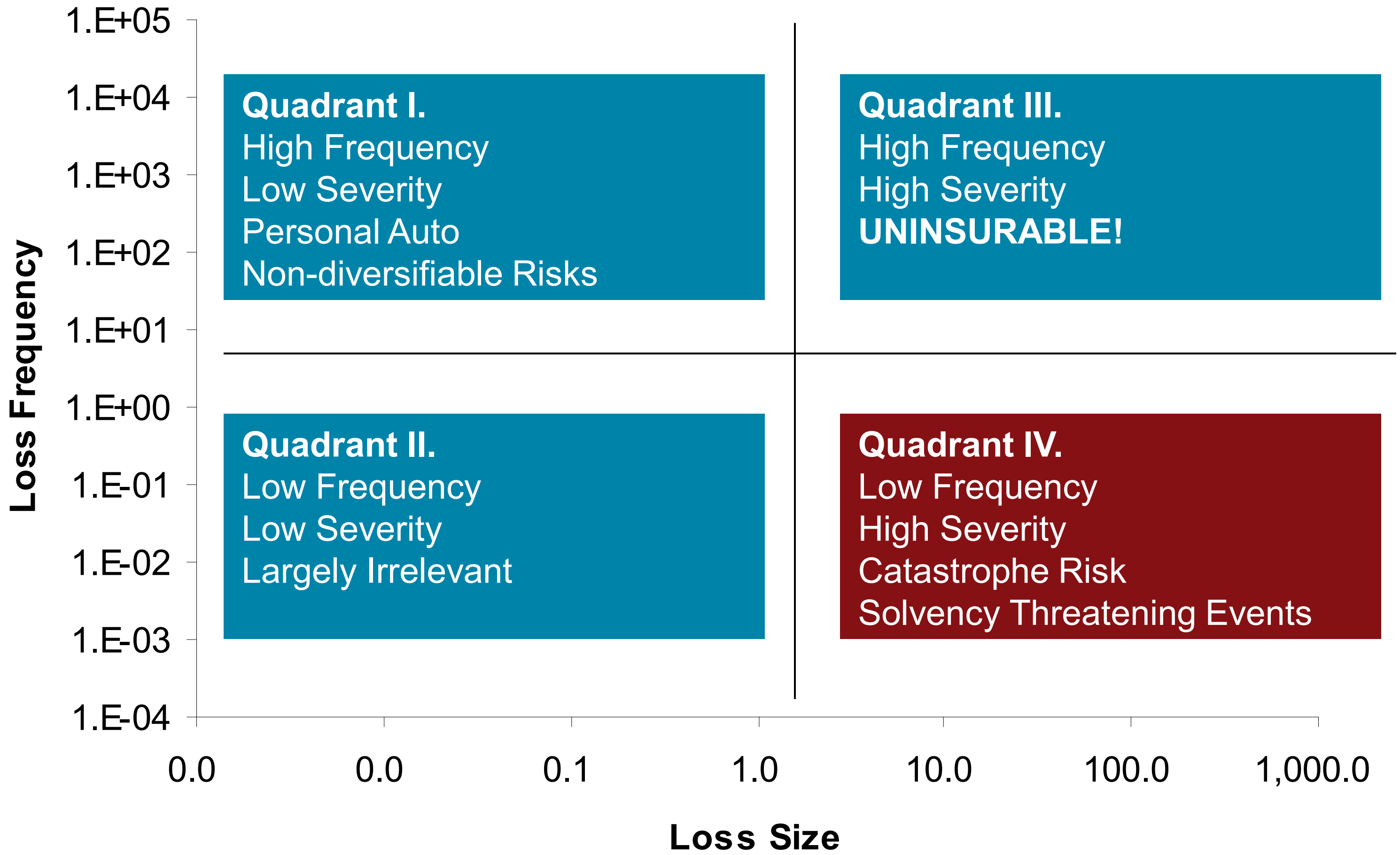
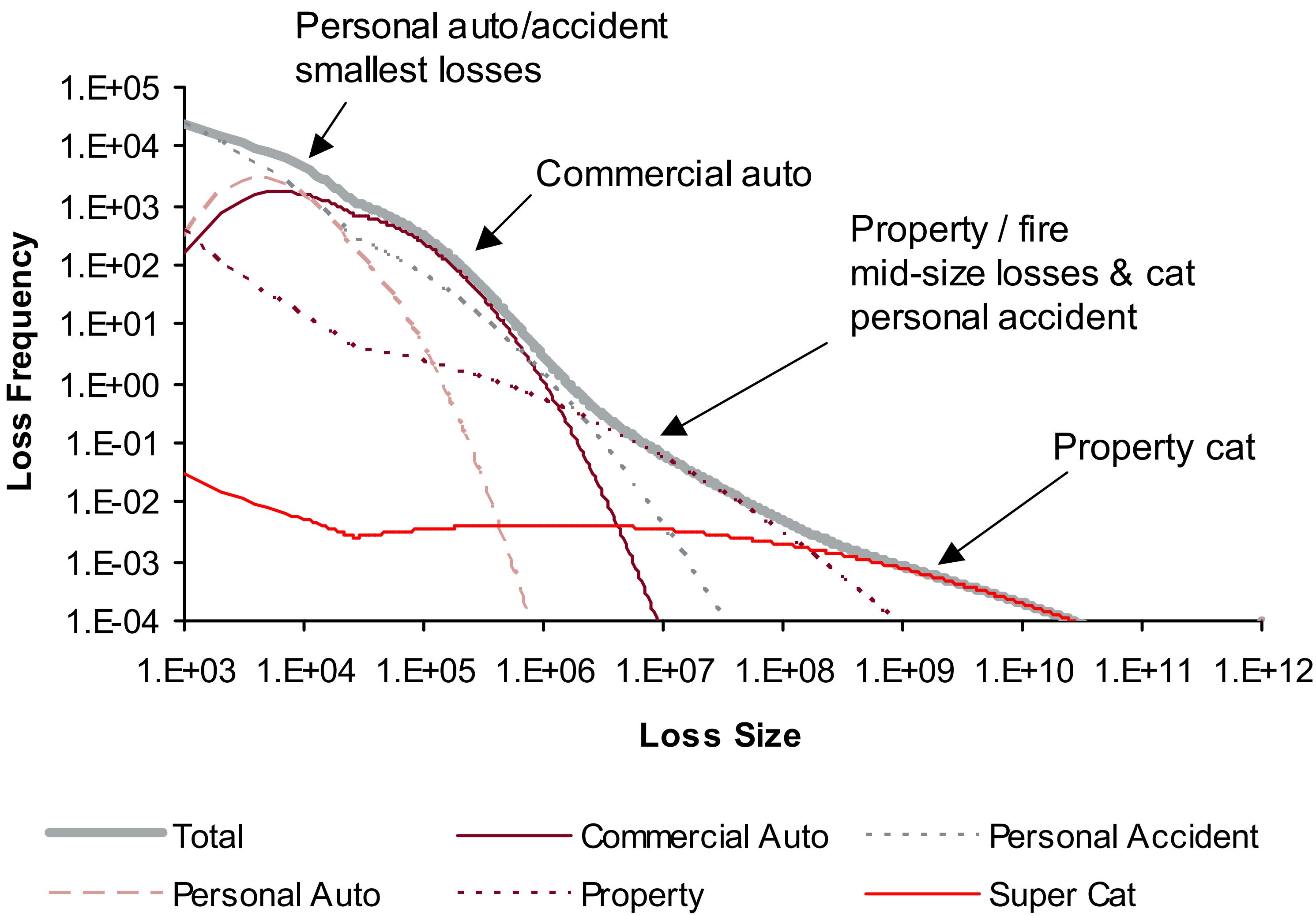
Four Quadrants of Risk



- Methodology applies to insurance risk and other risk sources such as asset, credit, mortality, reserve etc.

Thickest Tail Wins

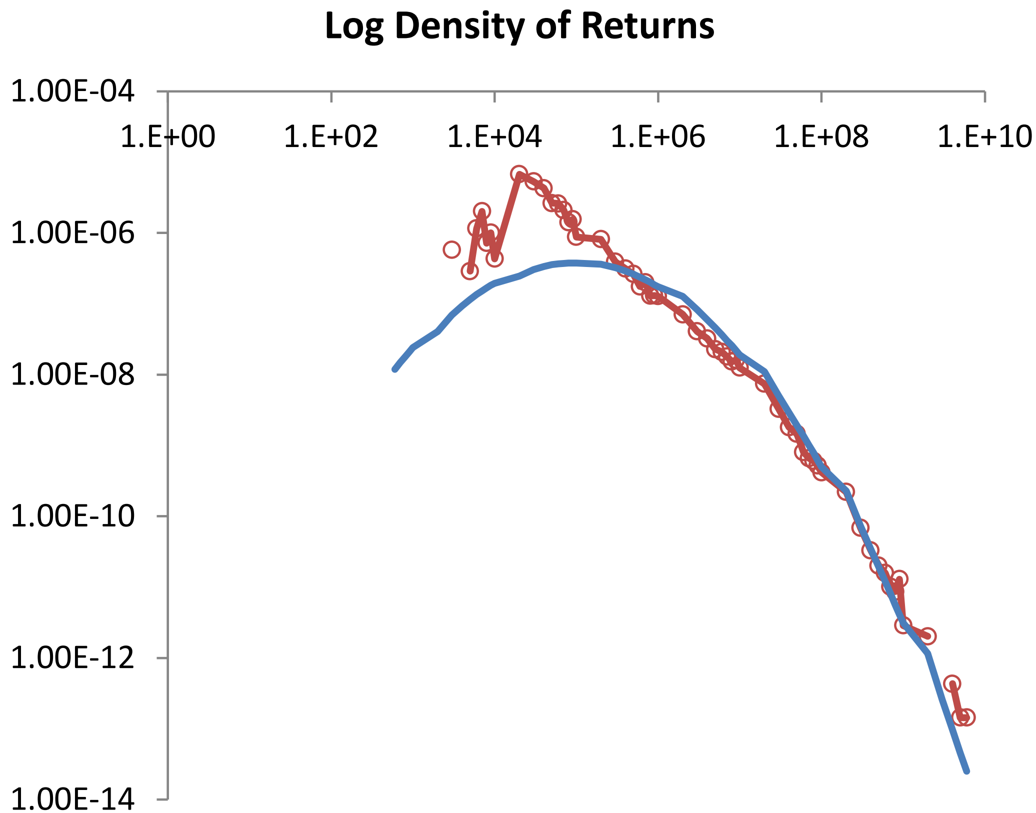


Univariate Data Analysis: Find the Thickest Tail

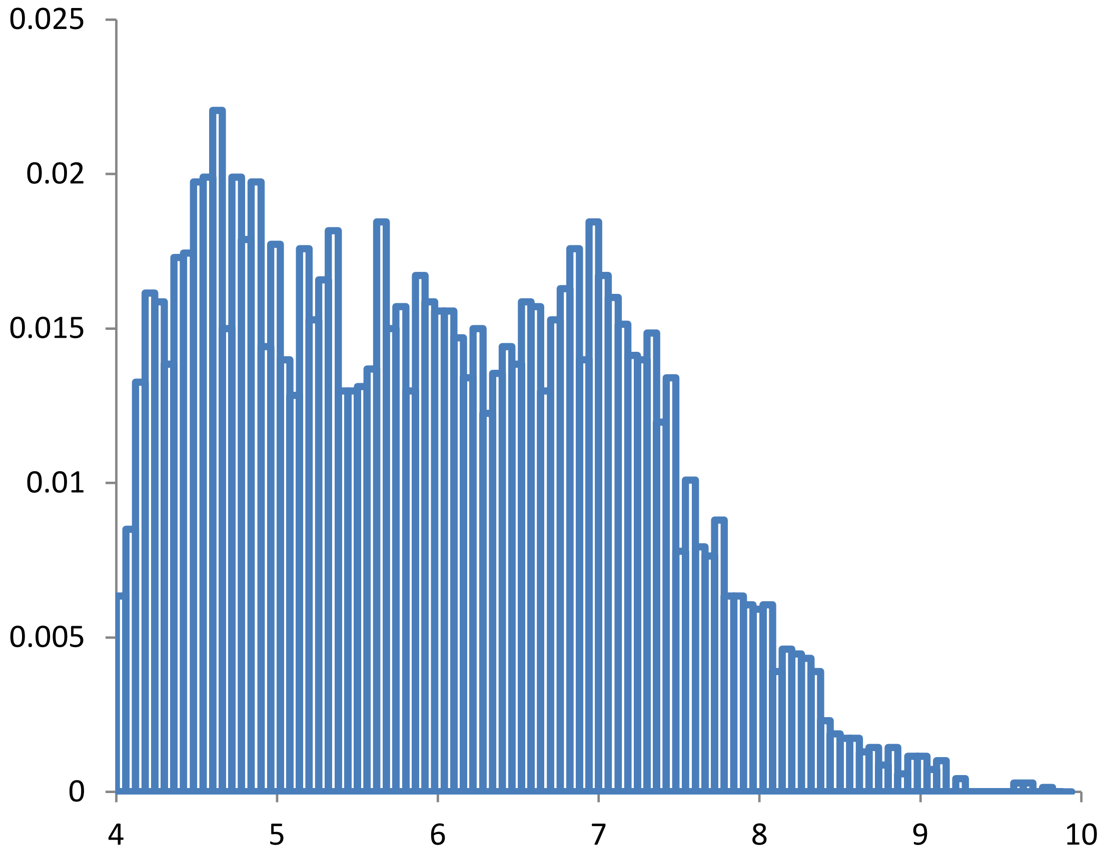
- SEC Class Action Settlements for D&O suits
- Stock price returns
- Scientific models: hurricane and earthquake sizes
- Rules of thumb for compound aggregate distributions

SEC Class Action Settlements for D&O Suits

Statistic	Value
Number of Observations	6,938
Proportion	100.0%
Average	22,196,111
Standard Deviation	139,360.0
CV	6.279
Skewness	21.954
Kurtosis	663.772
Max	307
99.9%	4,512
99.5%	8,098
99%	10,832
90%	24,555
Median	720,694
10%	32,143,784
1%	379,280,115
0.5%	735,735,614
0.1%	1,485,378,870
Min	5,573,022,885

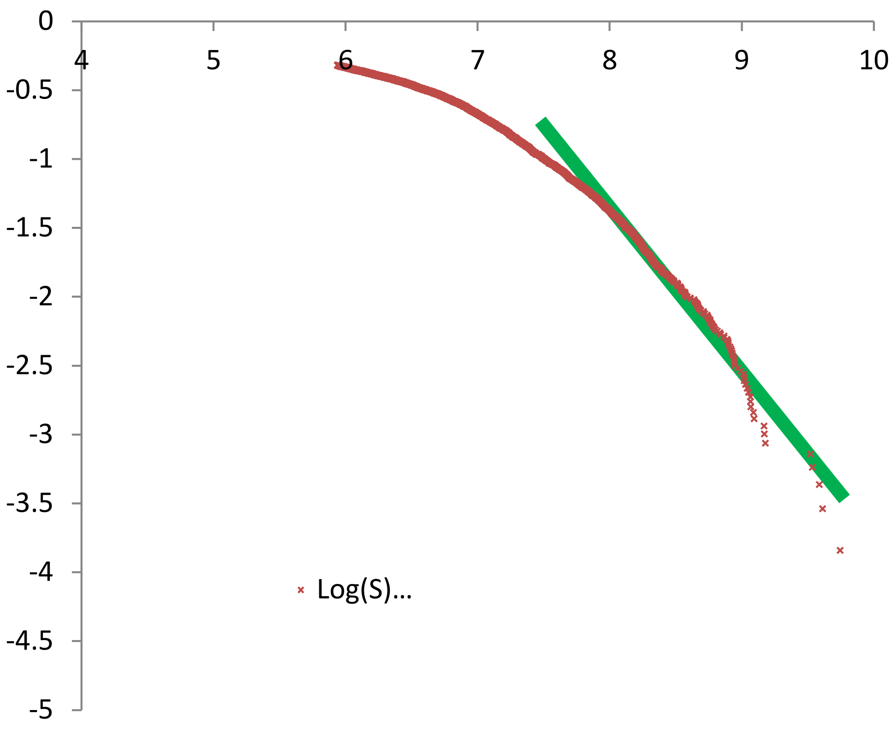


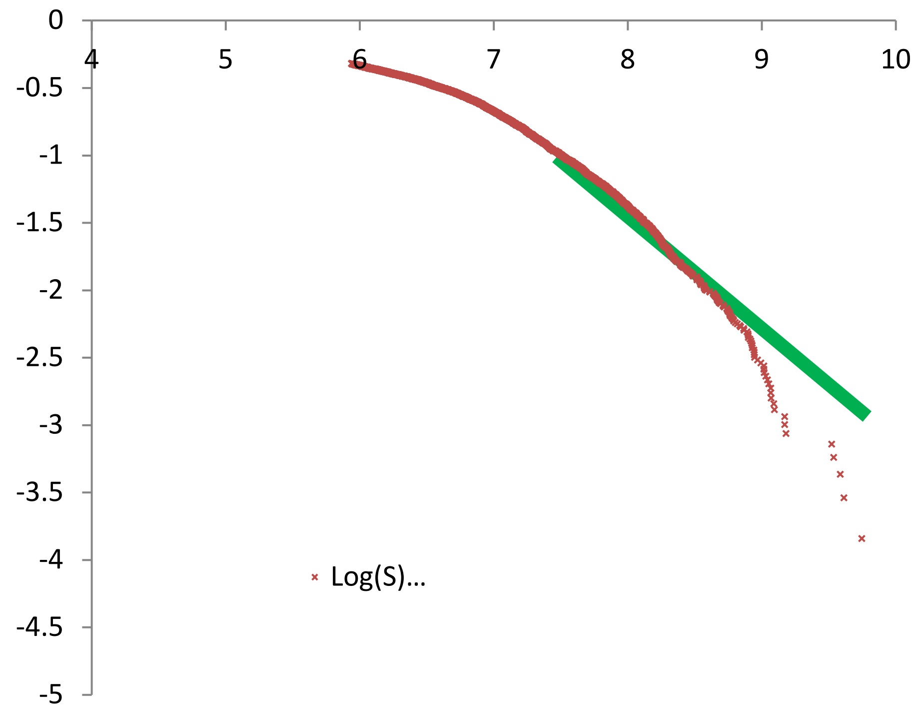
SEC Class Action Settlements for D&O Suits



- Mixture of Lognormal distributions indicated

SEC Class Action Settlements for D&O Suits





- Log survival function plotted against log(loss)
- Linear fit based on 5% of points (left) and 25% of points (right)
- Poor fit in both cases, indicating sub-polynomial severity distribution such as lognormal or mixture of lognormals

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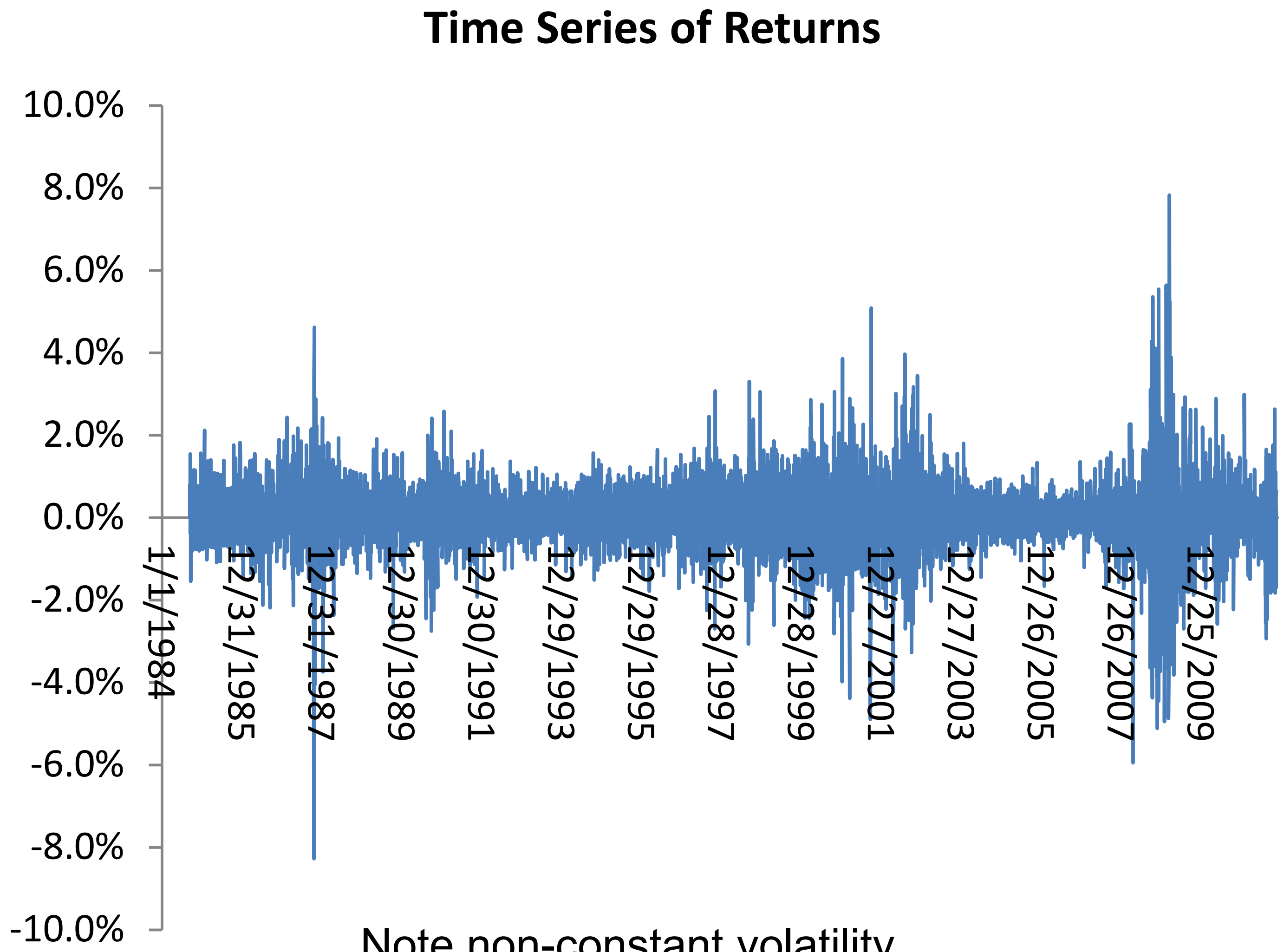
Stock Price Returns – GE Stock 1984 to Present

Statistic	All Data	Positive Days	Zero Days	Negative Days
Number of Observations	6,856	3,220	490	3,146
Proportion	100.0%	47.0%	7.1%	45.9%
Average	0.017%	0.601%		-0.577%
Standard Deviation	0.008	0.006		0.006
Skewness	-0.147	3.186		-3.453
Kurtosis	8.711	19.676		20.981
Max	7.823%	7.823%		-0.012%
99.9%	4.620%	5.355%		-0.015%
99.5%	2.747%	3.303%		-0.016%
99%	2.167%	2.874%		-0.027%
90%	0.879%	1.243%		-0.115%
Median	0.000%	0.445%		-0.398%
10%	-0.787%	0.115%		-1.187%
1%	-2.252%	0.026%		-2.939%
0.5%	-2.883%	0.017%		-3.820%
0.1%	-4.599%	0.014%		-4.946%
Min	-8.277%	0.012%		-8.277%

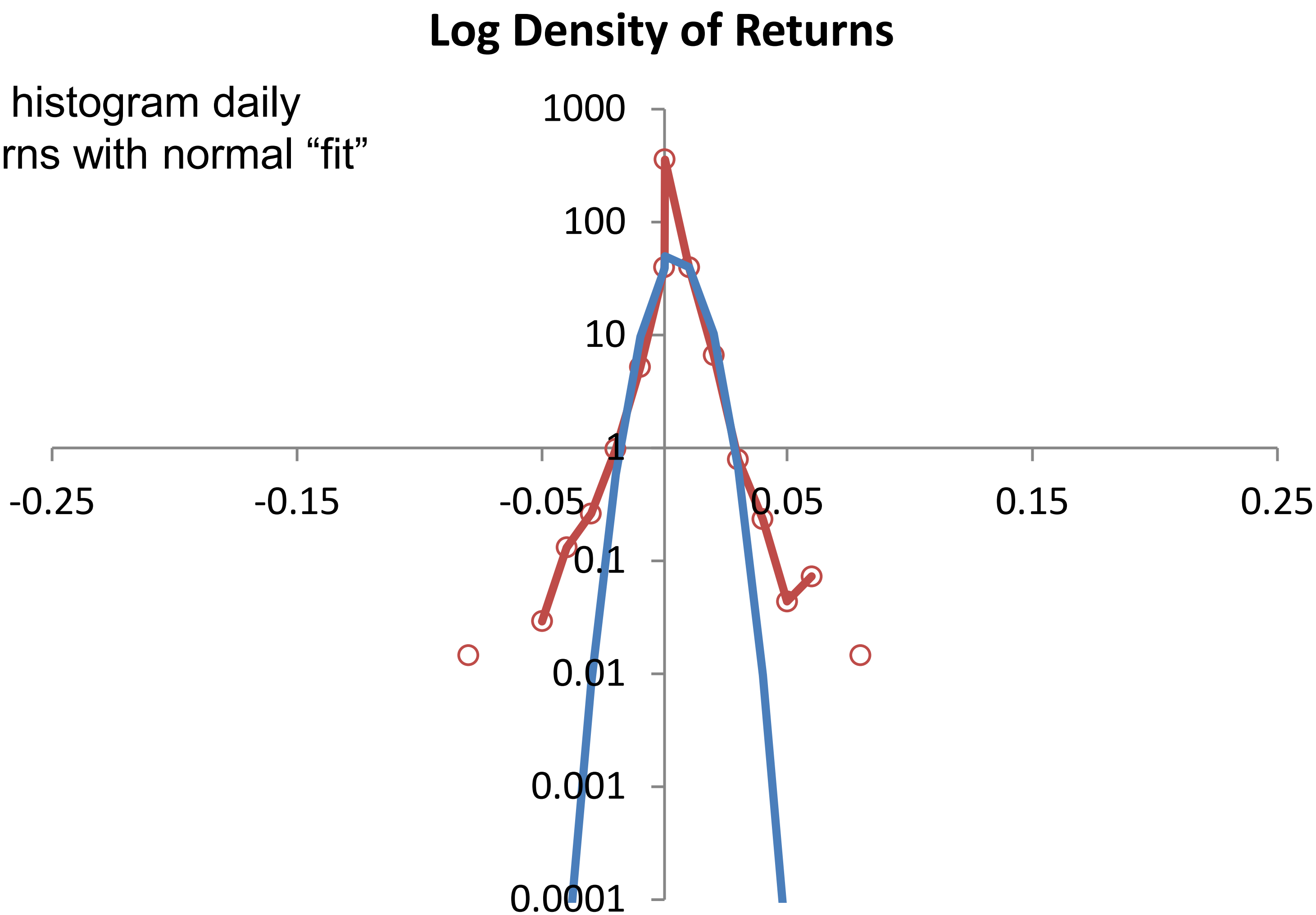
Stock Price Returns – GE Stock 1984 to Present



Note non-constant volatility

Stock Price Returns – GE Stock 1984 to Present

- Log histogram daily returns with normal “fit”



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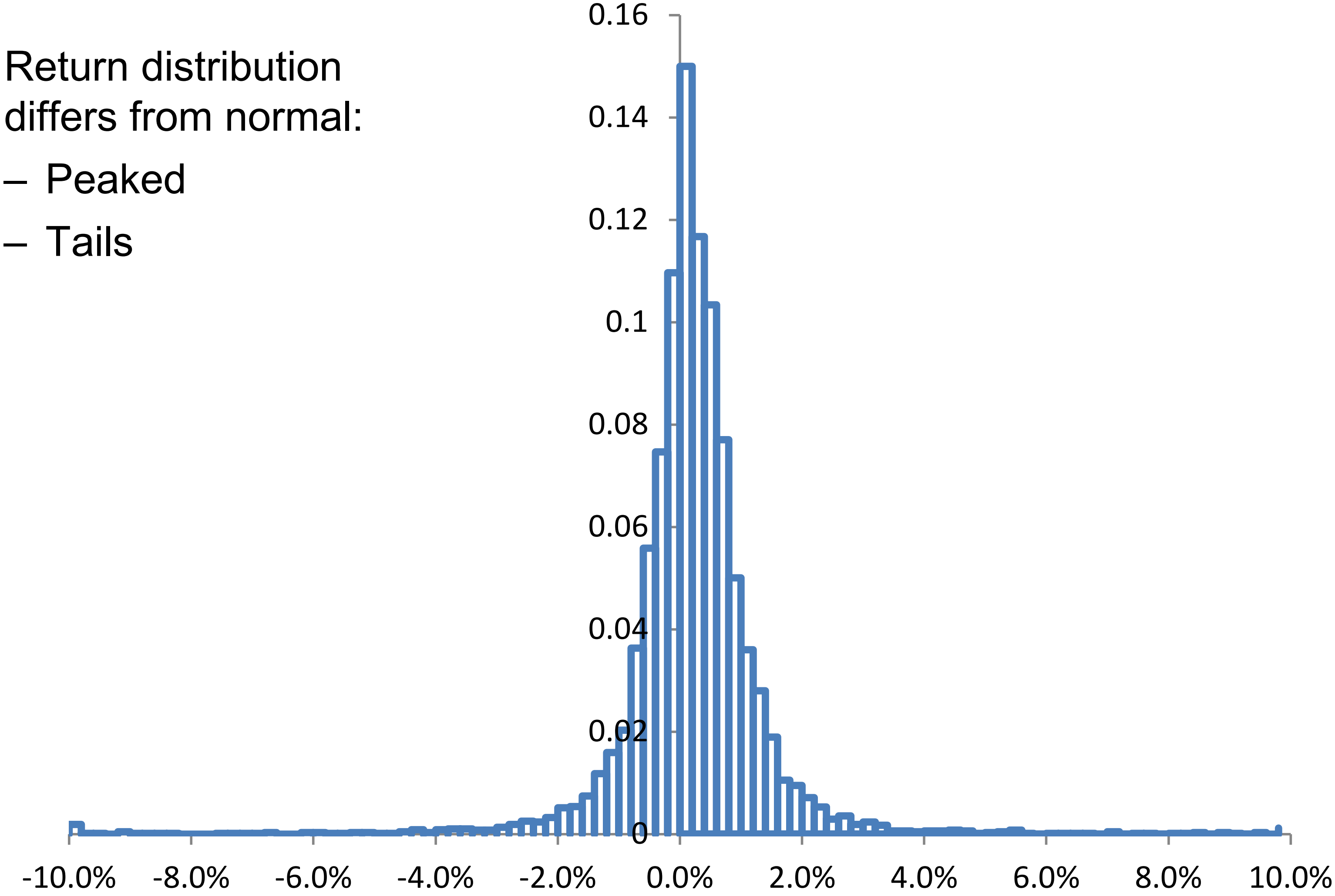
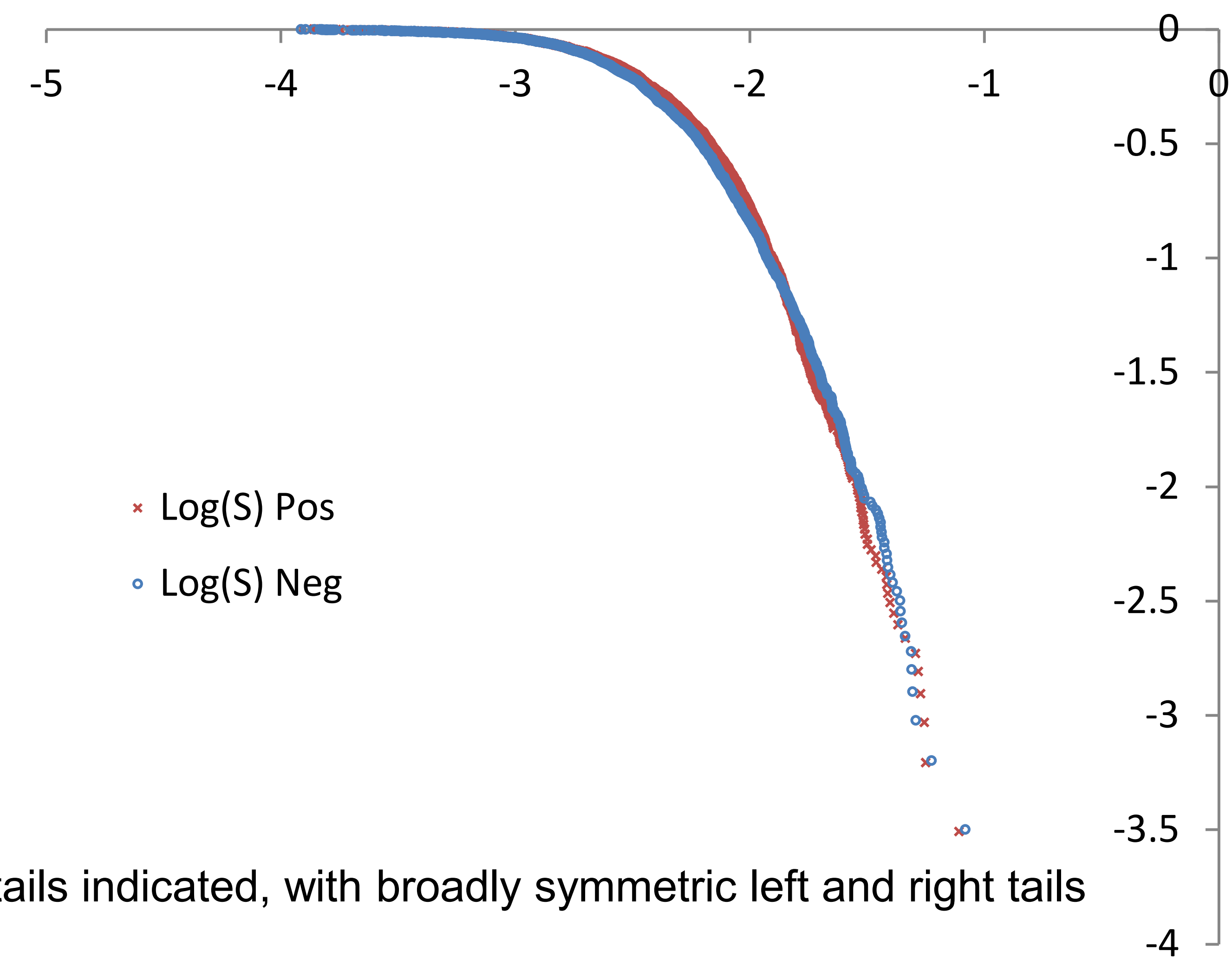
Stock Price Returns – GE Stock 1984 to Present

- Return distribution differs from normal:
  - Peaked
  - Tails

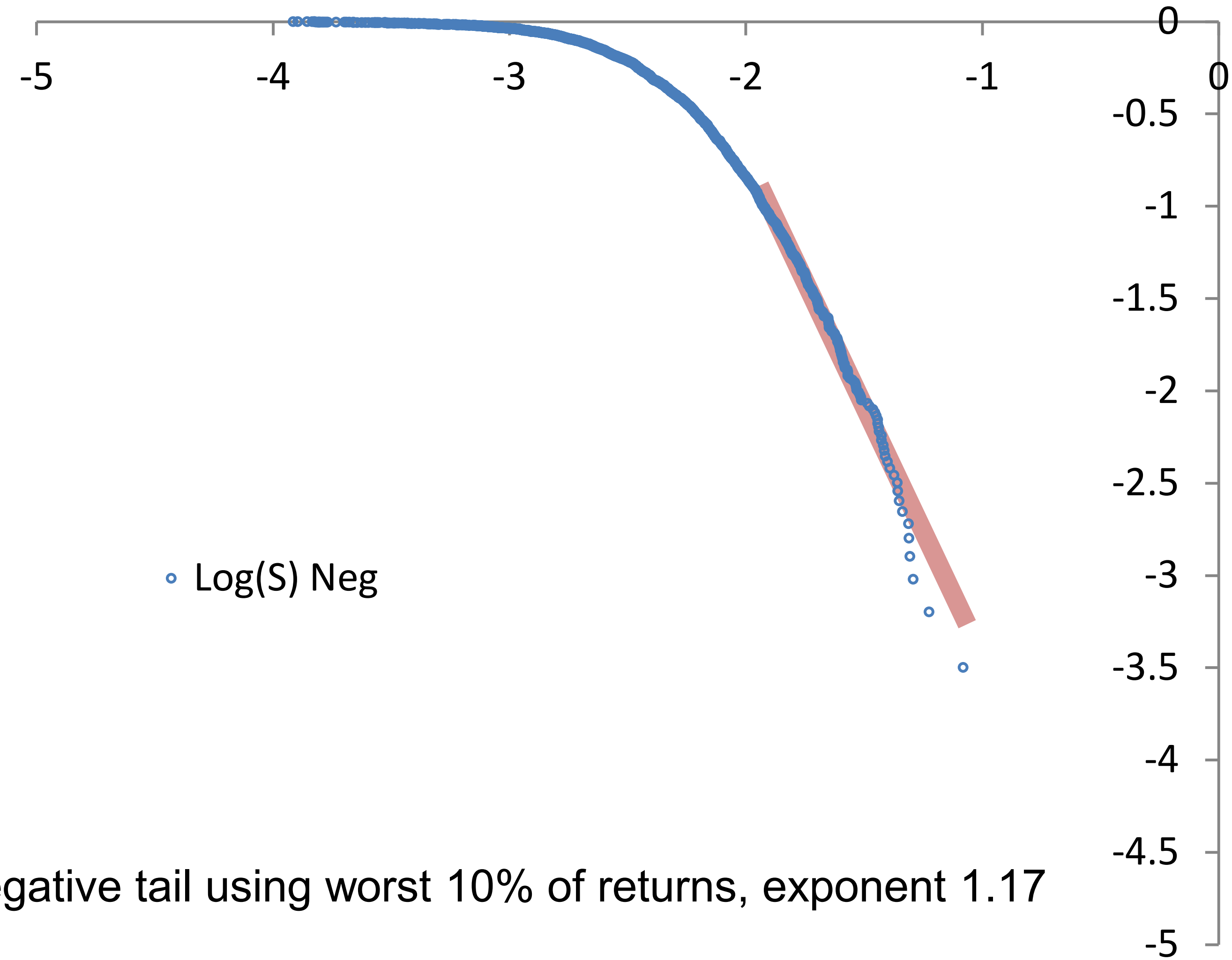


Stock Price Returns – GE Stock 1984 to Present



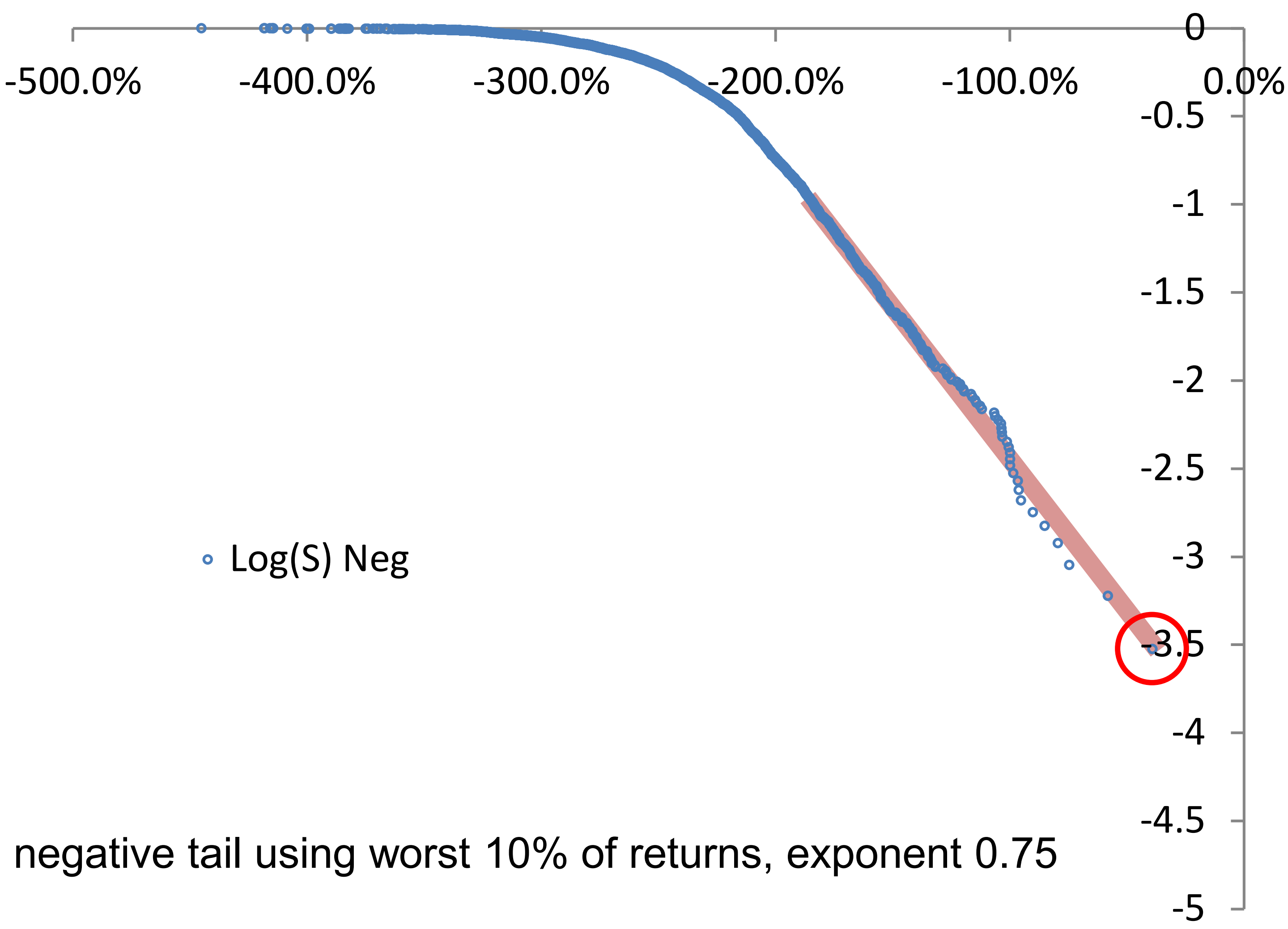
- Power tails indicated, with broadly symmetric left and right tails

Stock Price Returns – GE Stock 1984 to Present



- Fit to negative tail using worst 10% of returns, exponent 1.17

Stock Price Returns – Power Fit for AIG



- Fit to negative tail using worst 10% of returns, exponent 0.75

Science and Maximum Hurricane Wind Speed

- The **maximum potential intensity** (Kerry Emanuel) of a tropical cyclone is the theoretical limit of the strength of a tropical cyclone. It is computed using the following formula:

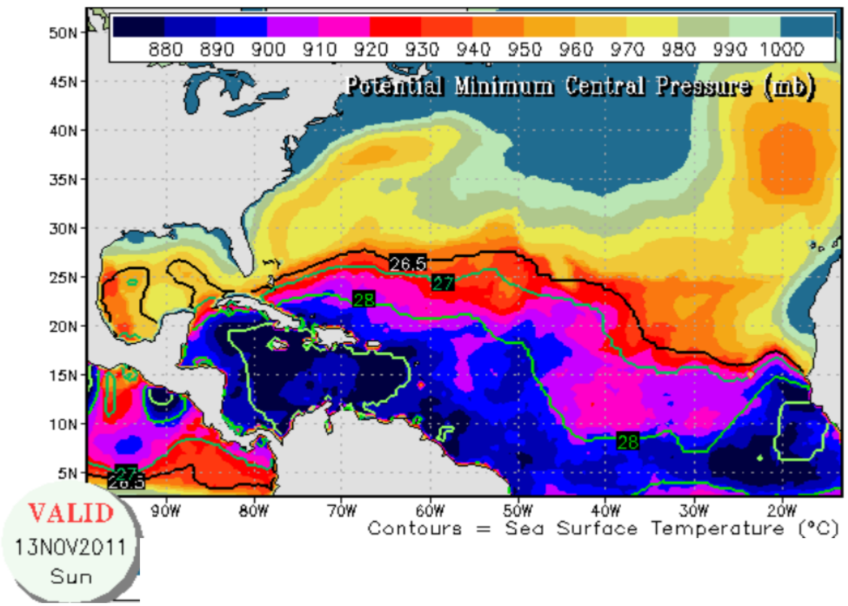
$$V = A + B * e^{C(T - T_0)}$$

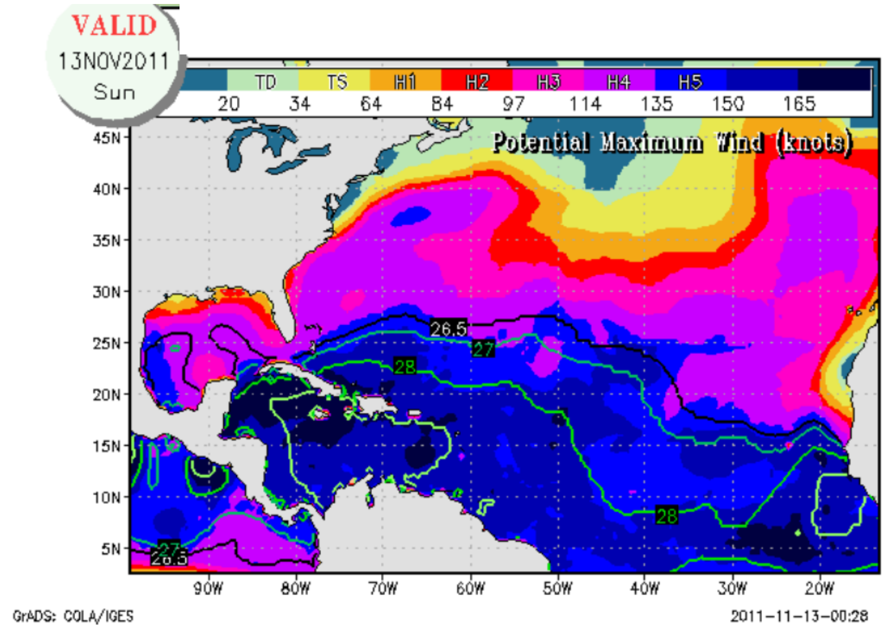
- Where
  - V is the maximum potential velocity in meters per second
  - T is the sea surface temperature underneath the center of the tropical cyclone
  - T_0 is a reference temperature, 30˚C
  - A, B and C are curve-fit constants

- When A = 28.2, B = 55.8, and C = 0.1813, the graph generated by this function corresponds to the 99th percentile of empirical tropical cyclone intensity data

Status in Atlantic on Sunday…

Atlantic Ocean:





- The maps display potential minimum pressure and maximum winds, calculated according to a method developed by Dr. Kerry Emanuel. Dissipative heating is handled according to a method described in Bister and Emanuel (1998).
- The maps are based on data from the 00Z global operational analysis from NCEP for the date shown on the plot.
- The left panel shows the potential minimum central pressure for a hurricane at any given location (in millibars). Only values less than 1000mb are shaded. Cyan squares indicate grid points where the algorithm failed to converge.
- The right panel shows the potential maximum wind speed expressed in terms of the type and severity of storm they would represent (*TD = Tropical Depression, TS = Tropical Storm, H1-H5 = Hurricanes of category 1-5 on the Saffir-Simpson scale*).

http://wxmaps.org/pix/hurpot.html

Accuracy of Any Formula Critical…

Statistic	Category 3	Category 4	Category 5
Max. simulated event	1,303	1,018	1,953
99%ile	396	374	1,386
50%ile	17	52	399
50% TVaR	81	144	725
Expected Loss	44	81	458

- What are the units?

- Similar approach for earthquakes says maximum intensity is about M 9.5
  - Fault length and slippage correlate strongly with magnitude
  - Earth's 'tectonic motion' energy budget; approx. 4 quakes a year above M 7.5, with a standard deviation of about 2; 1 above M 8, SD 1
- Model risk: of 7 prior journal publications considering the maximum magnitude of an event on the fault plane that ruptured during the Tohoku M 9 event, only 2 gave an upper limit greater than 9

Source: RMS 11 estimated industry gross loss, near-term including demand surge but excluding storm surge; critical Northeast US landfall

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Rules of Thumb

- For a compound aggregate distribution ($S_N = X_1 + \dots + X_N$, X_i iid severities and N independent frequency) if N is thick tailed and X is not thick tailed then severity is irrelevant

$$\Pr(S_N > x) \sim \Pr(N > x / E(X)) \text{ as } x \to \infty$$

Small severities are irrelevant

- If X_i are sub-exponential and sum $\Sigma(1+e)^n p_n$ converges for some e>0 then as x tends to infinity

$$\Pr(S_N > x) \sim E(N)\Pr(X > x)$$

Condition true for Poisson frequency

- = Don't Get Distracted by Aggregate VaR

Rules of Thumb

- If M_n and S_n are the partial max and partial sum of a series of iid thick tailed (sub-exponential) distributions then

$$\lim_{n \to \infty} \frac{\Pr(S_n > x)}{\Pr(M_n > x)} = 1 \text{ for all } x$$

- Largest element wins!

- Safe to ignore noise:

TVaR(Thick + Noise) = TVaR(Thick) + E(Noise)

provided TVaR(Thick) >> E(Noise)

Rules of Thumb

- Probability of a large loss from a sum of power law variables is approximately the sum of the probabilities from each component, and the sum is dominated by the thickest tail for large enough losses
- Thickest tail wins again, find minimum α
- New records
  - For thick tail distributions the new record is "of a different kind"
  - For thin distributions the new record is similar to the previous one
  - Example: wealth vs. height

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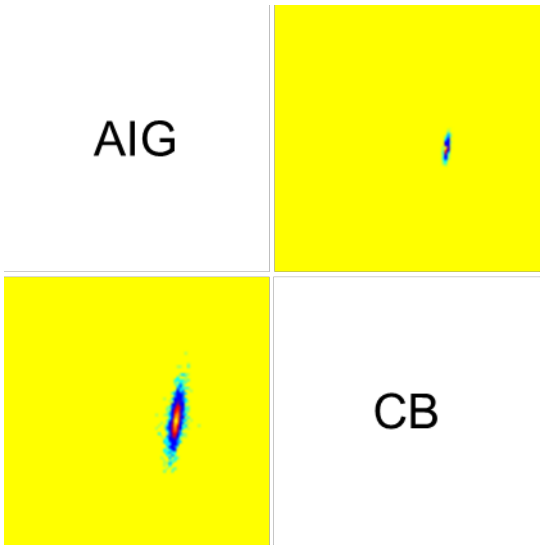
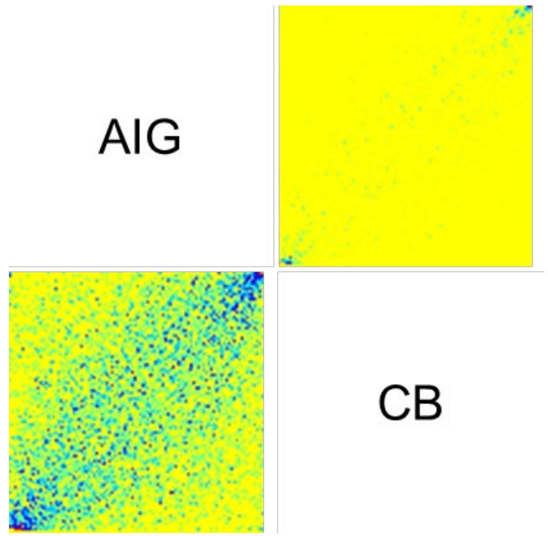
Visualizing Dependency

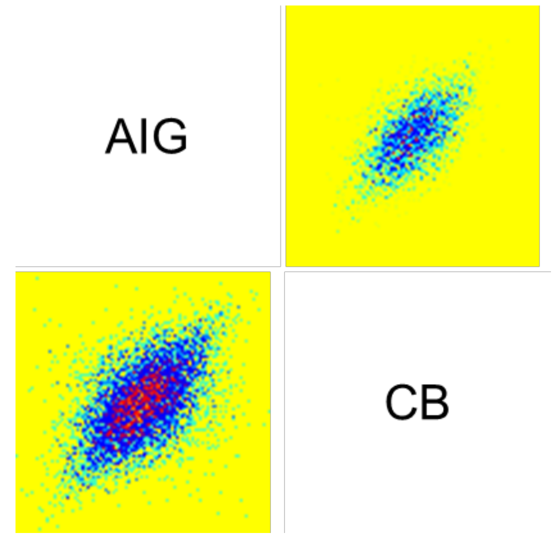
Actual Data: Daily Stock Returns, 1985-2011



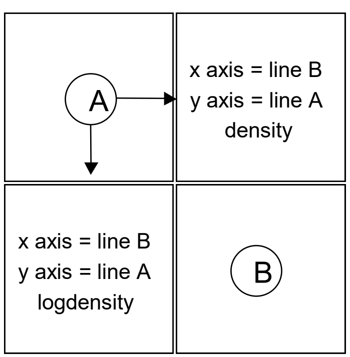
“Copula” View – Rank Returns



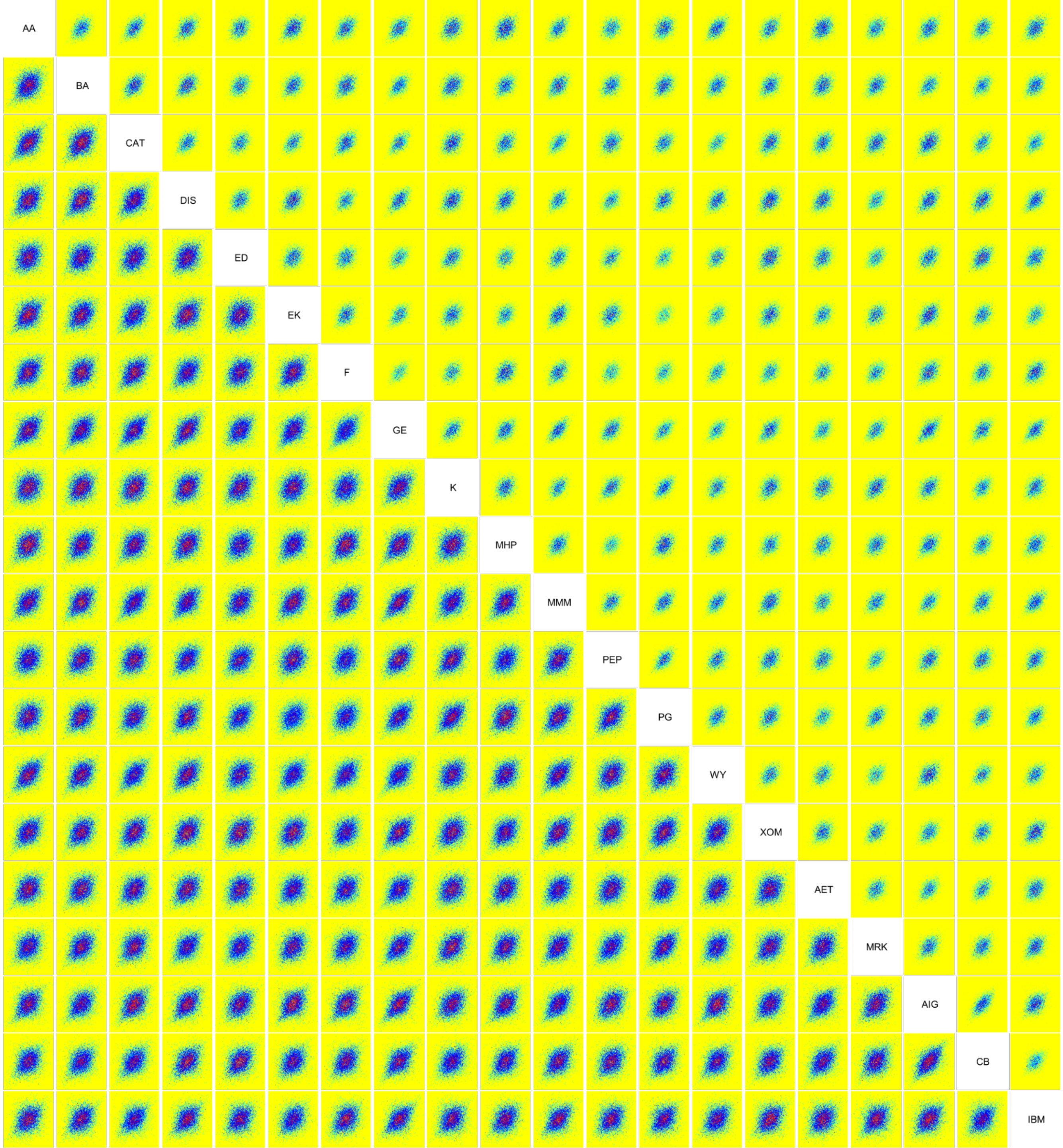
Normal Transformed Data



Key to Correlation Plots



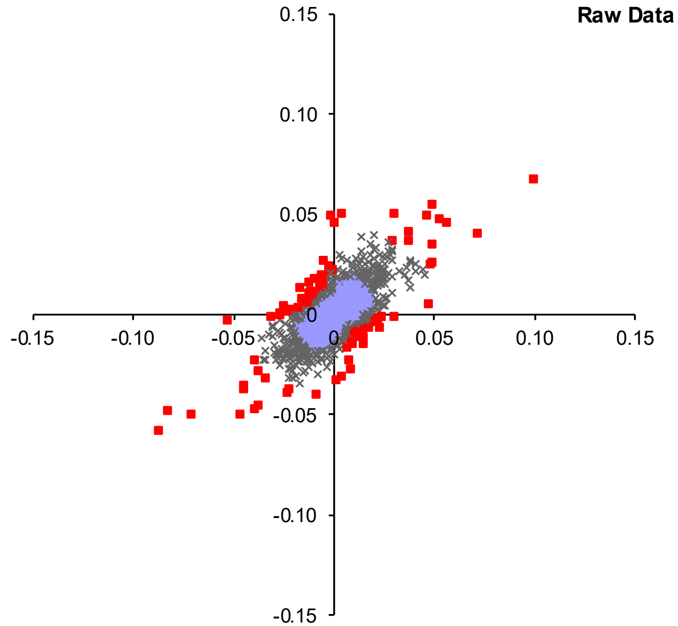
- Actual data subject to scaling issues
- Oft-seen “copula” view is not informative and does not appeal to intuition
- Normal transformed data shows extreme tail dependency clearly

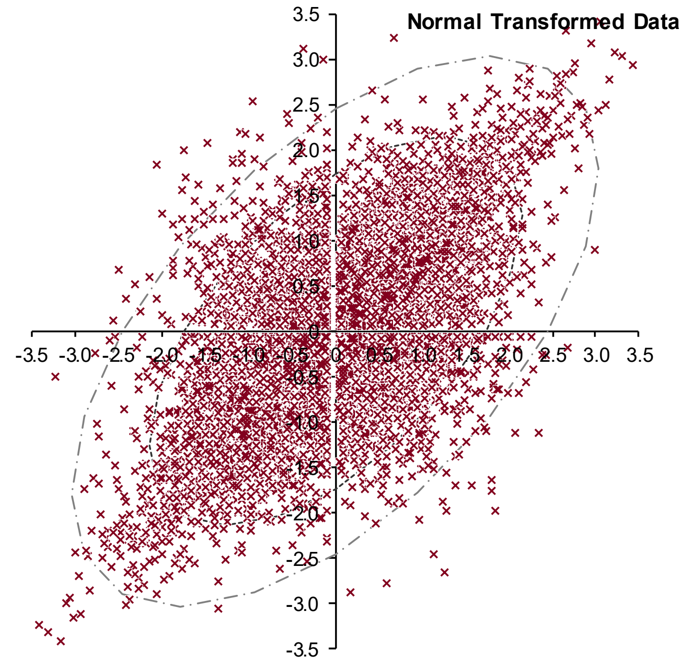


AA
BA
CAT
DIS
ED
EK
F
GE
K
MHP
MMM
PEP
PG
WY
XOM
AET
MRK
AIG
CB
IBM

Bivariate Dependency for Stock Returns
TRV vs. CB

TRV (x-axis) vs. CB | **6383 Annual Observations**





Association Summary

Linear Correlation, rho	64.3%
90% Confidence Interval	(63.1%, 65.5%)
Base Linear Correlation	62.4%
Extreme Linear Correlation (n=657)	65.9%
Rank Correlation	54.3%
Rank Correlation from rho	62.5%
Normal-Transformed Correlation	59.0%
Kendall Tau	39.1%
Rho from tau	57.7%
Outliers at 10% and 1% levels	10.3% and 1.5%

Univariate Summary

	TRV	CB
Mean	0.0002	0.0002
Min	-0.0872	-0.0583
Max	0.0988	0.0674
Std. Dev.	0.0081	0.0075
CV	3953.9%	3527.4%
Skewness	0.10	0.31
Kurtosis	13.71	7.64
90th percentile	0.8%	0.8%
99th percentile	2.2%	2.2%

Note: 1% outliers from normal copula marked in red. 10% and 1% and confidence intervals show on right.

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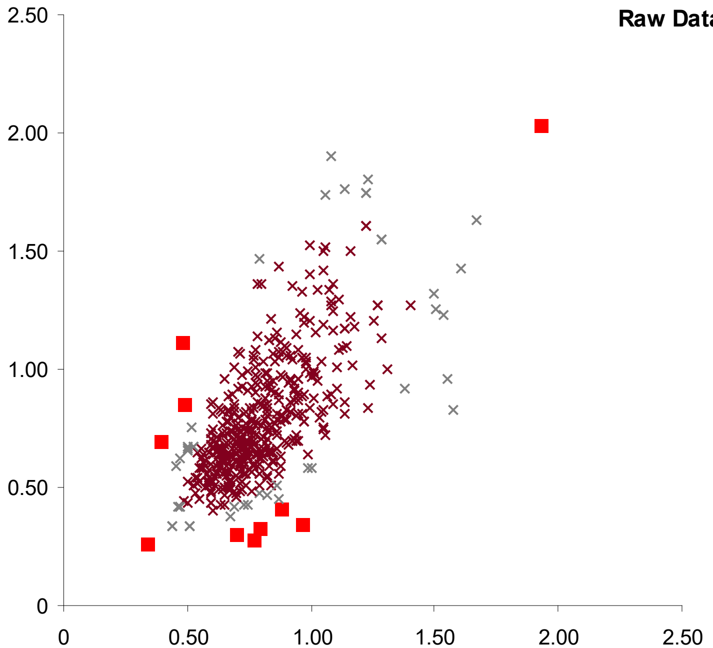
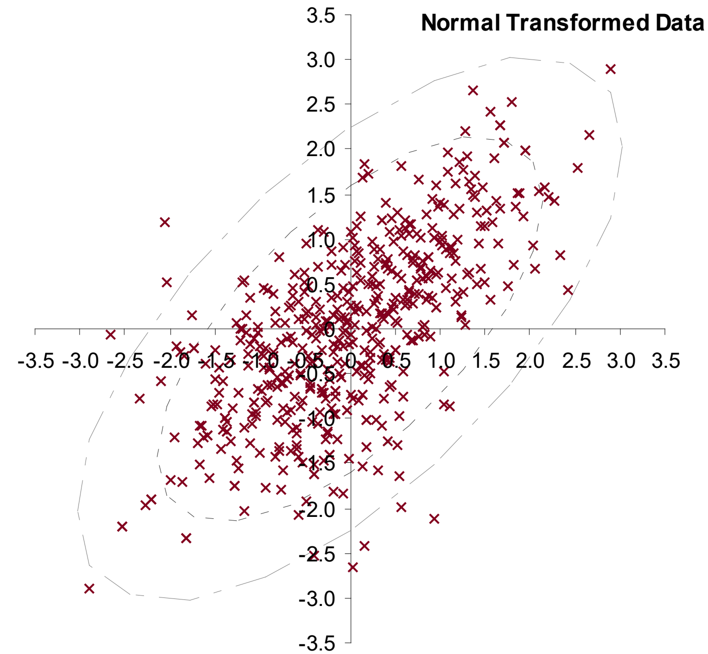
Bivariate Dependency for Insurance Losses
Commercial Auto Liability vs. Other Liability Occurrence

Commercial Auto Liability (x-axis) vs. Other Liability Occurrence, $100M premium threshold **514 Annual Observations**





Association Summary

Linear Correlation, rho	70.3%
90% Confidence Interval	(66.5%, 73.8%)
Base Linear Correlation	70.6%
Extreme Linear Correlation (n=52)	69.6%
Rank Correlation	70.4%
Rank Correlation from rho	68.6%
Normal-Transformed Correlation	66.9%
Kendall Tau	48.2%
Rho from tau	68.7%
Outliers at 10% and 1% levels	10.1% and 1.9%

Univariate Summary

	Commercial Auto Liability	Other Liability Occurrenc
Mean	0.7999	0.7842
Min	0.3381	0.2659
Max	1.9288	2.0328
Std. Dev.	0.2053	0.2767
CV	25.7%	35.3%
Skewness	1.35	1.33
Kurtosis	3.37	2.30
90th percentile	105.5%	115.3%
99th percentile	153.5%	174.4%

Note: 1% outliers from normal copula marked in red. 10% and 1% and confidence intervals show on right.

Dealing with Large Correlation Matrices

- Given n variables there are n(n-1)/2 correlations between them…so estimating all coefficients requires a substantial amount of data
  - Q = T/n = ratio of number of observations to number of variables

- Random Matrix Theory (RMT) helps estimate how much of an empirical correlation matrix is signal vs. noise

- RMT concept
  - Eigenvalues of correlation matrix greater or less than 1 show how far the multivariate distribution is from *spherical* (=independent)
    - Basis for principal components analysis
  - Distribution of eigenvalues of random matrices of a given size is known
  - Compare eigenvalues from empirical correlation matrices with the random matrix null hypothesis distribution
  - Larger Q corresponds to tighter null distribution around 1 (spherical)

Application to Insurance

Acc Year	CMP	Comm Auto	Home	Other Liab Occ	PPAuto	Work Comp
1992	112.0%	70.9%	165.1%	56.3%	77.8%	77.2%
1992	87.8%	77.5%	177.1%	100.5%	78.4%	83.2%
1992	105.1%	55.9%	103.3%	60.0%	82.7%	127.2%
1992	65.5%	79.1%	96.3%	32.8%	72.6%	73.6%
1992	77.4%	78.4%	108.3%	64.2%	81.8%	73.8%
1992	90.1%	71.3%	68.5%	51.3%	86.6%	77.8%
1992	126.9%	155.3%	117.2%	96.1%	86.6%	66.5%
1992	103.4%	77.6%	166.5%	108.5%	85.7%	82.6%
1992	135.4%	84.9%	220.3%	65.1%	103.3%	75.1%
1992	451.9%	68.8%	519.7%	62.0%	82.7%	73.5%
1992	87.7%	86.7%	95.8%	102.2%	80.7%	78.4%
1992	104.3%	73.7%	118.8%	51.2%	72.6%	84.5%
1992	84.1%	67.7%	75.2%	57.3%	74.4%	63.6%
1992	62.6%	61.6%	66.5%	34.1%	64.0%	67.7%
1992	82.6%	64.5%	105.3%	[illegible]	[illegible]	66.5%
[illegible]	64.3%	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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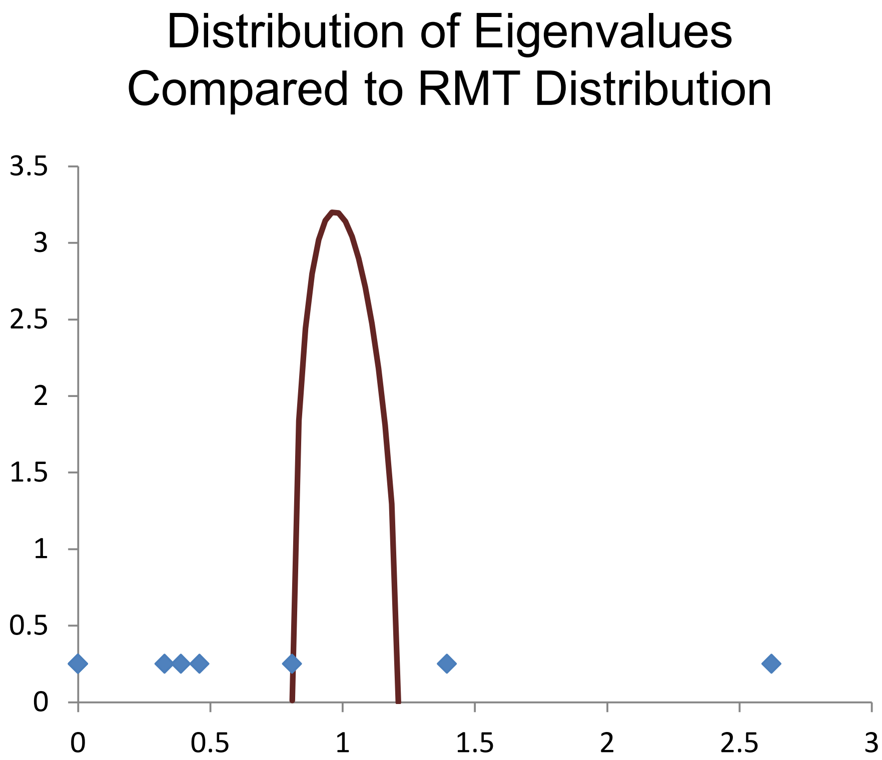
Application to Insurance



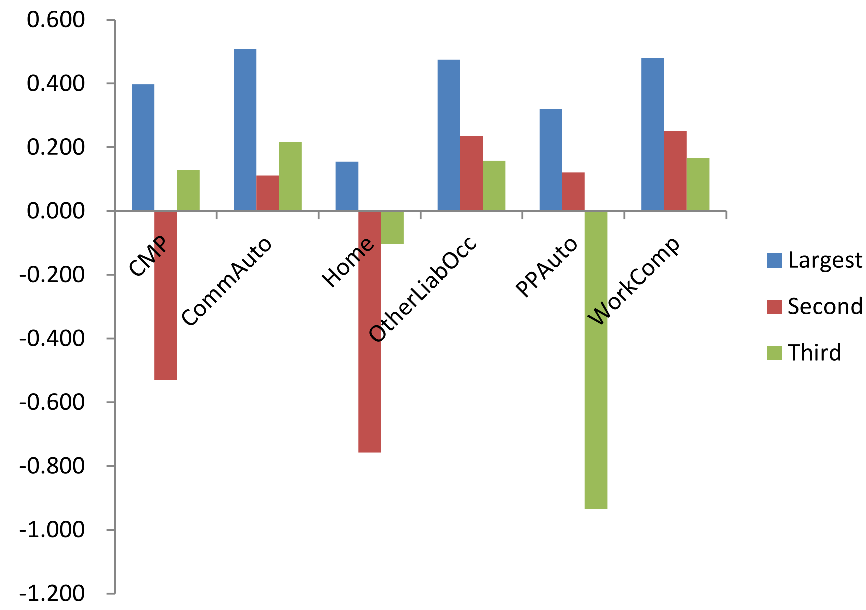
Statistic	Value
N	6
T	596
Q=T/N	99.333
q	0.010
Lambda:	
min	0.809
max	1.211

- Six US lines of business, gross basis, premium over $25M
- 1992 to 2010
- One “large” eigenvalue at 2.6, and one above range of RMT

Application to Insurance

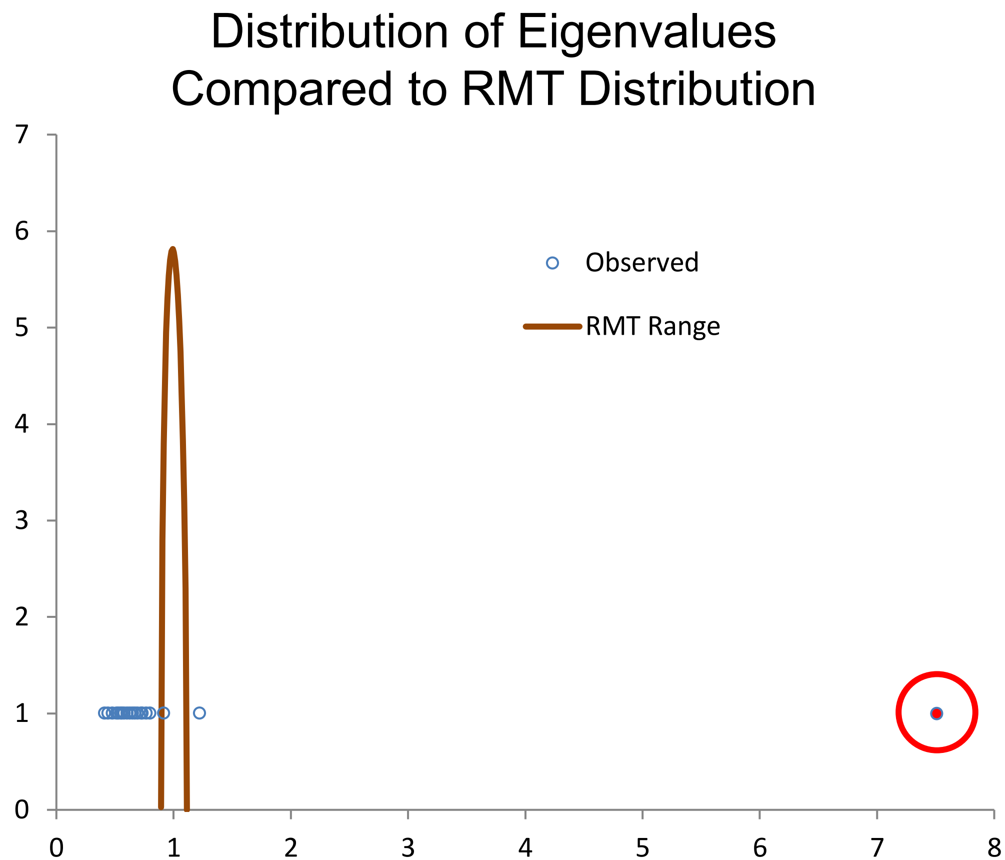
Largest Eigenvectors

Line	Largest	Second	Third
CMP	0.397	-0.530	0.129
CommAuto	0.509	0.112	0.216
Home	0.155	-0.757	-0.104
OtherLiabOcc	0.474	0.236	0.158
PPAuto	0.321	0.121	-0.935
WorkComp	0.481	0.251	0.166
Eigenvalue	2.621	1.394	0.809



- Eigenvector corresponding to largest eigenvalue is the market: approx. equally weighted across all lines
- Second largest picks up property vs. liability (CMP and Home)
- Third largest picks out personal lines vs. commercial lines

Application to Stock Price Returns



Statistic	Value
N	20
T	6638
Q=T/N	331.9
q	0.003
Lambda	
min	0.893
max	1.113

- Note Q = observations to variables much greater
- Tighter theoretical distribution under RMT null hypothesis

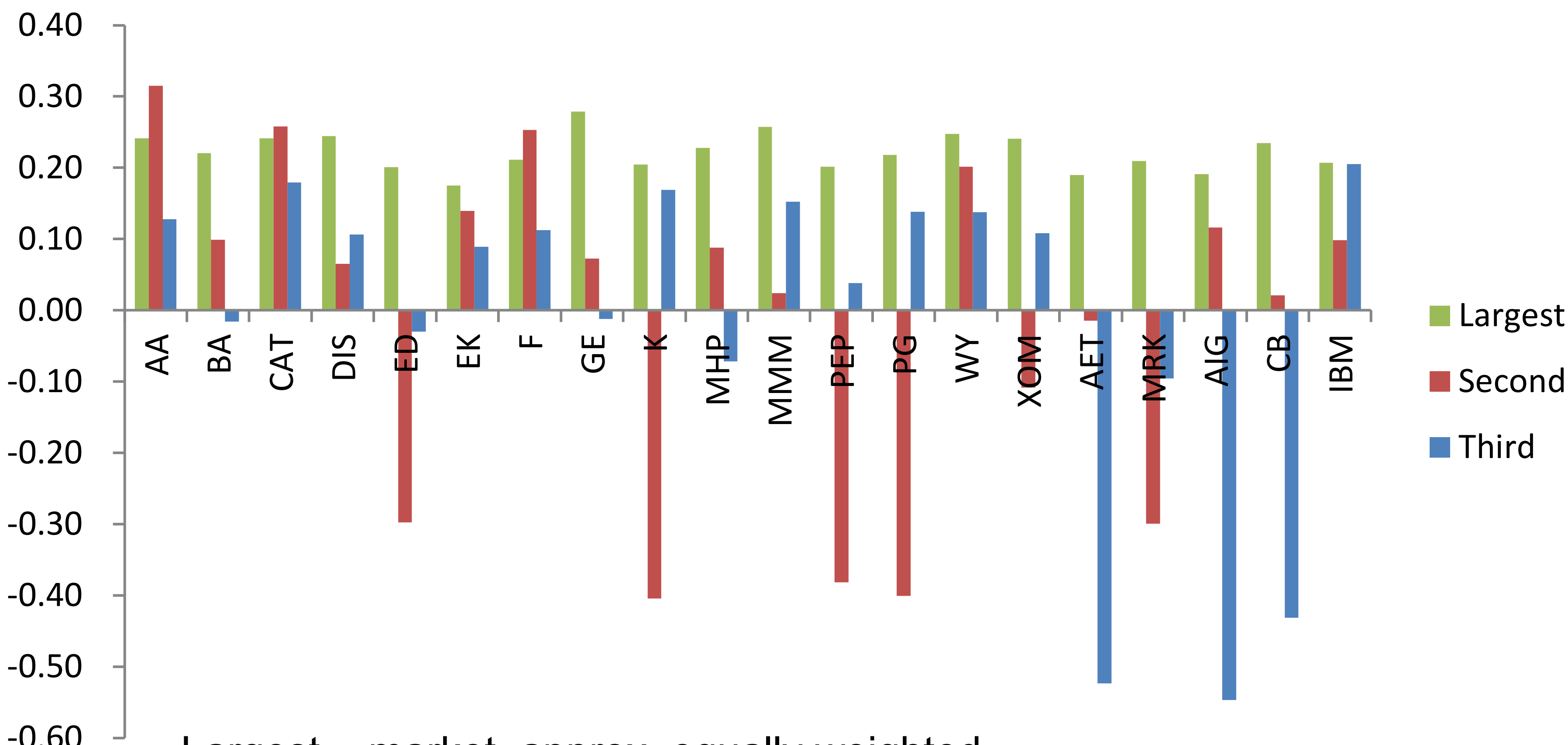
- Twenty large cap stocks
- Daily return data 1985 to 2011
- The market return huge outlier

Application to Stock Price Returns

Largest Eigenvectors

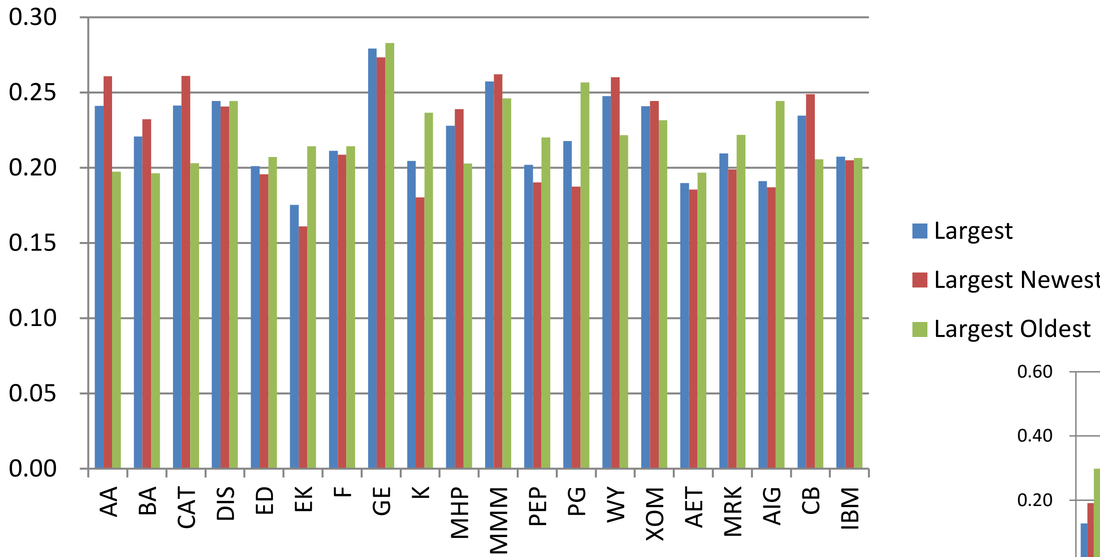
Ticker	Largest	Second	Third
AA	0.24	0.32	0.13
BA	0.22	0.10	-0.02
CAT	0.24	0.26	0.18
DIS	0.24	0.07	0.11
ED	0.20	-0.30	-0.03
EK	0.18	0.14	0.09
F	0.21	0.25	0.11
GE	0.28	0.07	-0.01
K	0.20	-0.40	0.17
MHP	0.23	0.09	-0.07
MMM	0.26	0.02	0.15
PEP	0.20	-0.38	0.04
PG	0.22	-0.40	0.14
WY	0.25	0.20	0.14
XOM	0.24	-0.11	0.11
AET	0.19	-0.01	-0.52
MRK	0.21	-0.30	-0.10
AIG	0.19	0.12	-0.55
CB	0.23	0.02	-0.43
IBM	0.21	0.10	0.21
Eigenvalue	7.51	1.22	0.92

Application to Stock Price Returns



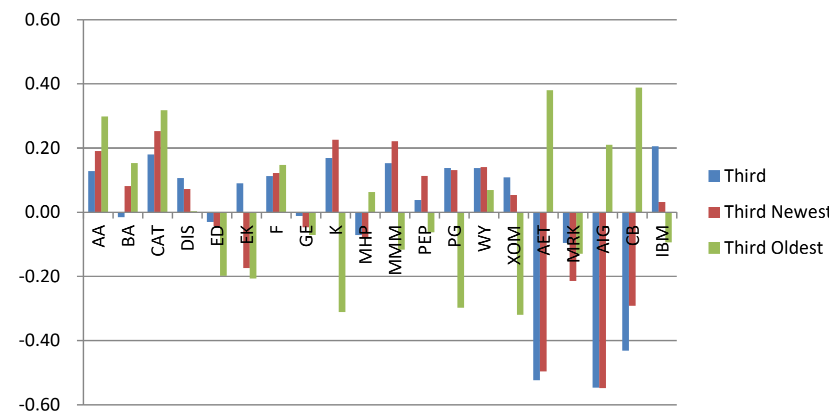
- Largest = market, approx. equally weighted
- Second = consumer staples: Edison, Kellogg, Pepsi, Proctor & Gamble, Exxon, Merck
- Third largest hard to interpret, skewed towards financials

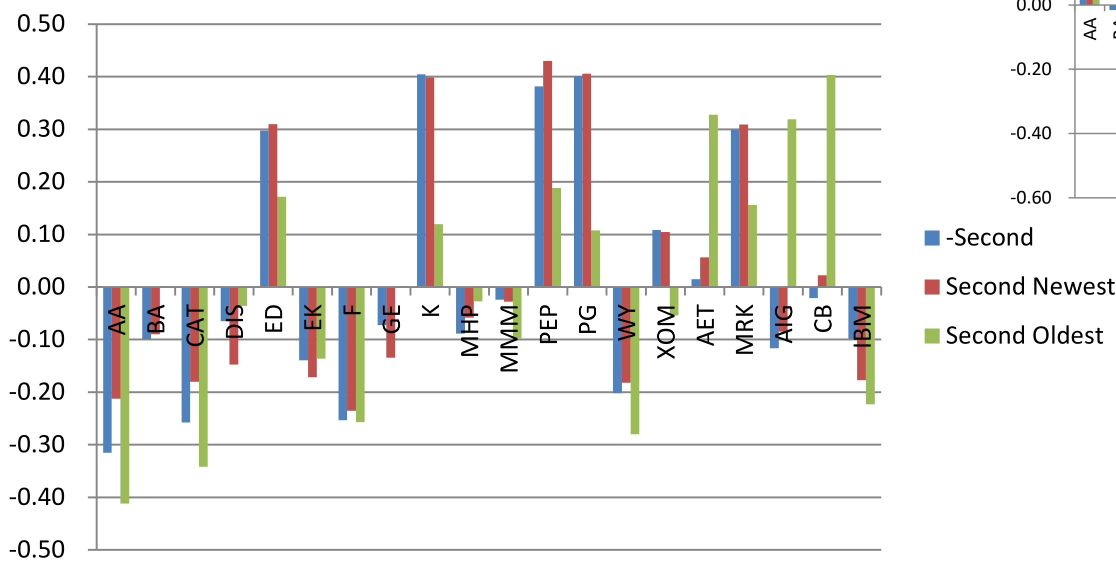
Consistency of Results



Consistency of Eigenvalues

Oldest Half	Newest Half	All Data
1.031	0.878	0.917
1.080	1.305	1.223
7.724	7.581	7.509





AON BENFIELD

Why? Drivers of thick tails and/or high dependency?

- Dependency or thick tails: are the two effects separable?
  - Catastrophe risk = thick tailed severity or correlated frequency?
  - Stock portfolio return: do you even consider individual stock returns? …and if you do, what correlation glue do you apply?

- Look at informative pictures, not copula diagrams
  - May need to apply suitable normalizing transformations

- Do you really have trustable dependencies?
  - Use RMT to test

- Scientific models can give physically-driven limits to tail thickness
  - Auto accidents and kinetic energy…
  - Workers compensation, medical: limits of human body!
  - Quantum electrodynamics vs. hurricane model: are limits useful?

Why? Drivers of thick tails and/or high dependency?

- Crowding and herding behaviours; management actions and animal spirits
- Strategy correlation
  - 1987 portfolio insurance crisis
  - LTCM: illiquid positions
  - 2007 hedge fund unwinding: strategy correlation
- Model correlation
  - Correlation through S&P model risk = CDO/CDS all move at once
  - RMS v. 11.0
- Systemically risky and structural instability
  - Banks: borrow short / lend long will always be risky
  - Trading strategies with positive feedback loops (pricing off from model)
- Management
  - Insurance pricing cycle and reserving decisions
  - Understanding problem not equivalent to finding a solution

Summary

- **Work in progress**
  - Suggested relevant tools and results
  - Barely scratched surface
  - No EVT, extreme value theory
- **Minimum α**
  - Find the thickest tail
  - Look for power laws as especially dangerous
- **Maximum Eigenvalue**
  - Test your correlations and ignore the noise
- **Why?**
  - Understanding drivers can suggest important behavioural changes
  - All working in a behavioural environment: management drives the cycle
  - While the music plays you've got to keep dancing…